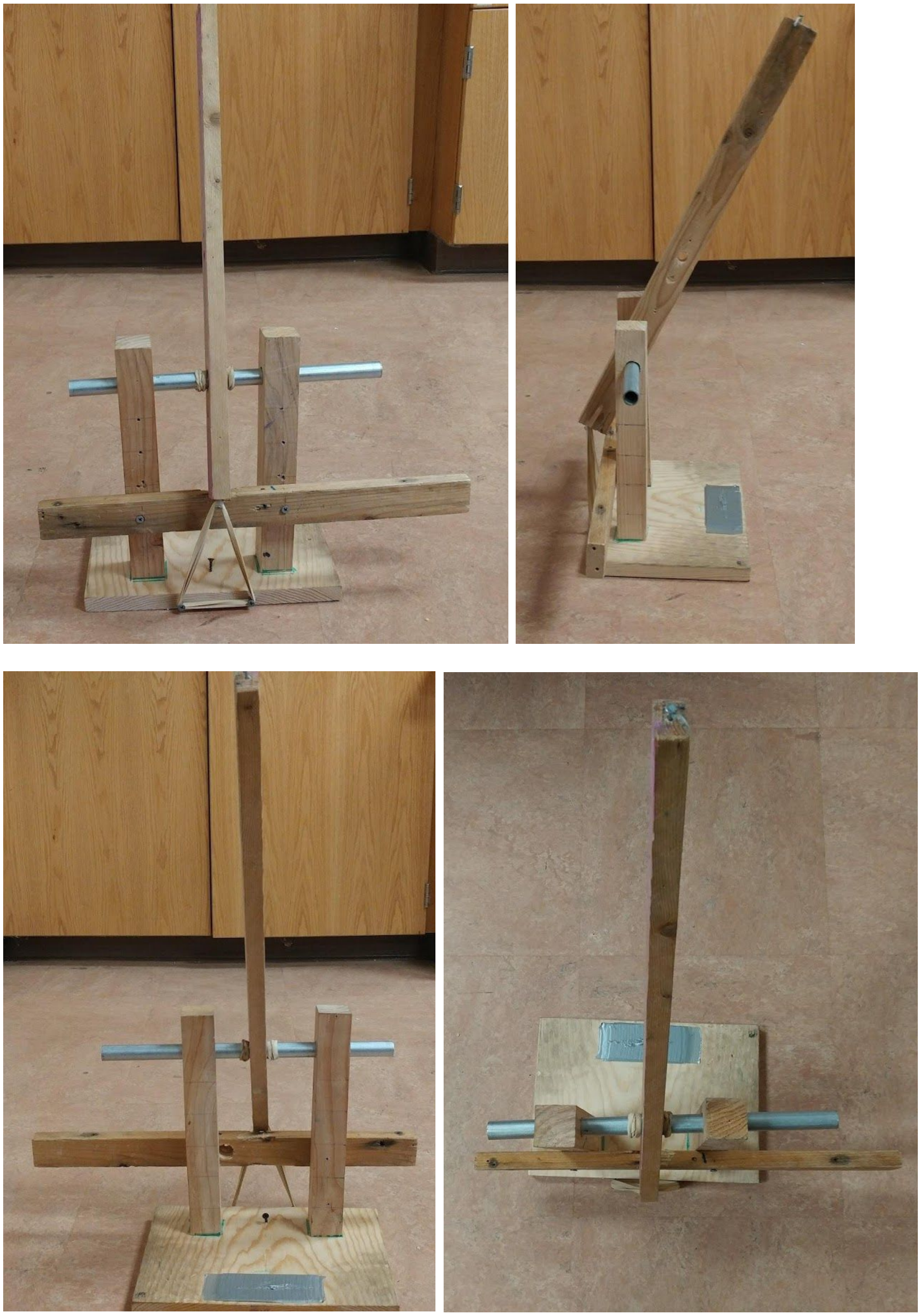
Andersen Teoh - Trebuchet Proof of Efficacy

Images.



Starting from top left rotating clockwise; front view, side view, back view, top view

Trebuchet Description

Our trebuchet consists of two 28 cm by 3.8 cm by 3.8 cm softwood legs with a 11 cm gap in between them, standing on a 28 cm by 19.7 cm by 2.2 cm softwood base. There is a 2 cm in diameter hole on both legs 23 cm up from the base with a 34 cm long aluminum axle 1.5 cm in diameter going through it. There are 5 holes drilled on the front of the trebuchet at heights of 8 cm, 9.5 cm, 12 cm, 16 cm, and 20 cm, with a 47 cm by 4 cm by 2 cm softwood stopper positioned at 9.5 cm. The firing arm is a 56 cm by 4 cm by 2 cm piece of softwood with a 2 cm in diameter hole drilled at 12 cm from the front of the arm. It has a screw to hold the rubber bands on the front end, and a flat head nail tilted 60º upwards on the opposite end. There are two screws in the front of the trebuchet in the base, 8 cm apart, both 10 cm from the sides of the base, and both 1.3 cm up from the ground. 8 size 64 rubber bands are hooked onto the two base screws and the screws on the firing arm. A 9.68 gram clay projectile on an 8 inch long piece of string with a loop at the end is placed onto the firing arm's nail. The long end of the trebuchet is then pushed down by a person and released to fire. Our trebuchet fires on average from 35-45 meters, however our largest distance was 52 meters.

Modifications

- 35º angle of release (9.5 cm high stopper)
    - A 35º angle of release would make the projectile fly the farthest because theoretically, a 45º angle would go the farthest, but in real life, accounting for air resistance which would decrease the horizontal velocity, a more horizontal angle would increase that velocity and make it go farther. On our trebuchet, that meant putting a wooden stopper at a height of 9.5 cm to stop the firing arm.
- 9.86 gram clay ball
    - The weight of 9.86 grams is perfect, because force is equal to mass times acceleration, if the mass is too light, it won't have as much force, and

won't go as far, however, the projectile cannot be too heavy as well because then the arm won't have enough force to throw it.

- 8 inch string
  - An 8 inch string maximizes the rotation the projectile can make and thus increases the distance without letting the ball rub against the ground and create friction.
- 1:4 ratio of arm
  - A 1:4 ratio of the arm is the perfect amount as the larger the ratio is, the more the force will be multiplied, but the load end will not be able to support the effort end if the ratio is too large, so a 1:4 ratio is just right.
- 56 cm length of arm
  - Depending on how large the trebuchet is, the length of the arm has to be long enough that the 1:4 ratio actually works and it has enough force to fire, but short enough that it allows the rubber bands to be pulled to their full potential.
- 8 rubber bands
  - The more rubber bands you can stack onto the trebuchet, the more potential energy, and in turn, kinetic energy, it has. However, there cannot be too many rubber bands as the wood of the trebuchet might break because of all the force.
- Size 64 rubber bands
  - Size 64, or the thickest, longest rubber bands work the best because the larger and thicker they are, the more potential energy can be obtained from one rubber band.
- Spacers
  - These helped our trebuchet quite a lot because our rubber band spacers on either side of the firing arm prevented the arm from moving side to side while firing, increasing the amount of force output in the forward direction instead of shaking around.

Clear Paragraph

The purpose of our trebuchet was to figure out what is the best placement of a stopper to stop the swinging arm, thus changing the angle of release. We figured out that the ideal location for the stopper is 12 cm from the base on our trebuchet, or a release angle of 35º on other trebuchets, placing the arm at an angle of 55º. We found this by putting a stopper at the distances of 4 cm, 8 cm, 12 cm, 16 cm, and with no stopper. At 4 cm, it fired from 9-11m, from 8 cm, it fired from 12-13m, from 12 cm it fired from 14-22 m, and with 16 cm, it fired from -1-6 m. Finally, without a stopper, it fired from 11-13 m. Using this, we found that a stopper at a distance of 12 cm from our base, or a release angle of 35º, launched our projectile the farthest. This makes sense, because we already know that the ideal release angle is 45º, however that is in a perfect world. In the real world, air resistance decreases the horizontal velocity, so a more horizontal release angle would have more horizontal velocity and travel farther.

Calculations

- Mass of Projectile: 9.86 grams or 0.00986 kg
  - We weighed the projectile and found the mass
- Horizontal Distance: 41 meters
  - Out of many firings, the average distance seemed to be about 41 meters
- Time in Air: 2.94 seconds
  - We got a stopwatch and timed the distance that went 38 meters
- Vertical Distance: 14.835 meters
  - Time = ½ (acceleration due to gravity)(time)^2
  - Time = ½ (9.8 m/s^2)(1.47 s (because only rising/falling))^2
  - Through this formula, we found that our trebuchet throws our projectile about 14.835 meters into the air. The high distance was expected because when we fire our trebuchet, we can’t even see the projectile firing.

- Horizontal Velocity: 13.95 m/s
  - Vertical velocity = distance/time
  - Velocity = 41 meters/2.94 seconds
  - Our trebuchet fires high to make up for its relatively low horizontal velocity. To find the horizontal velocity, we need the rate of distance in a period of time. The horizontal distance of 41 meters is covered in 2.94 seconds, leading to a velocity of 13.94 m/s
- Vertical Velocity: 28.812 m/s
  - Vertical velocity = (acceleration)(time)
  - Velocity = (9.8 m/s^2)(2.94 s)
  - The vertical velocity of our trebuchet is much faster than what would be expected. The high arc helps it achieve a maximum distance.
- Total Velocity: 32.01 m/s
  - Total velocity = a^2 + b^2 = c^2
  - Velocity = (13.95 m/s)^2 + (28.812 m/s)^2 = total velocity^2
  - You find the total velocity by imagining a right triangle, with the two sides connected to the right angle are the vertical and horizontal velocity, and the hypotenuse is the total velocity. By using the Pythagorean Theorem, you can find the total velocity.
- Angle of Release: 35º
  - We can find the angle of release quite easily, as we find the angle of the firing arm at its final point on the stopper, which is 55º, find the angle of the end of the arm which is 90º, and since a triangle has to be 180º, the angle of release is 35º. We can also find this by getting a protractor and measuring it.
- Spring Constant: 30.25 newton/meters
  - Spring constant = force/distance
  - The spring constant measures the strength of the rubber bands. We put a 1 kg weight on a rubber band, and found out that it extended 0.324

meters. To find the force you use F=ma or F=(1 kg)(9.8 m/s^2) which is 9.8 newtons. Using the equation for spring constant, 9.8 N/0.324 meters is 30.25 newton/meters. So each one of our 8 rubber bands has a spring constant of approximately 30 newton/meters.

- Initial Spring Potential Energy: 43.506 joules
  - PEspring = ½ (spring constant)(distance displaced by rubber bands)^2
  - PEspring = ½ (30.25 newton/meters)(1.696 meters (it is the displacement of all 8 rubber bands))^2
  - If we insert the spring constant of 30.25 newton/meters and the displacement of all 8 rubber bands, where a singular one is 0.212 so all 8 would be 1.696 meters, the result is that our potential energy is 43.506 joules.
- Kinetic Energy: 5.05 joules
  - Kinetic energy = ½ (mass)(total velocity)^2
  - Kinetic energy = ½ (0.00986kg)(32.01 m/s)^2
  - A 9.86 gram ball and a total velocity of 32.01 m/s inserted into the equation makes the kinetic energy in the the ball 5.05 joules.
- Percent Energy Converted: 11.6%
  - Transferred energy = KE/PE
  - If we divide our kinetic energy (5.05 joules) by our potential energy (43.506 joules) we can find out the actual percentage of energy converted from our rubber bands to our release. An insanely large amount of energy was lost in our trebuchet, I believe that is because of friction and air resistance throughout the trebuchet, and the side to side swing of the firing arm also affects the energy transfer as well.

Superiorities of our Trebuchet

- **Fires exceptionally far, from 35 meters all the way up to 52 meters**
- **Has a much higher arc than other trebuchets, enabling it to have a longer air time**
- **Has an extremely large amount of PE and if modified correctly, could fire insanely far**
- **Small, portable, and very easy to carry around, unlike some other trebuchets in the class**